

## Assignment 12

### 13.1

1. Use the set of productions to show that each of these sentences is a valid sentence.

- a) *the happy hare runs*
- b) *the sleepy tortoise runs quickly*
- c) *the tortoise passes the hare*
- d) *the sleepy hare passes the happy tortoise*

3. Show that the *hare runs the sleepy tortoise* is not a valid sentence.

4. Let  $G = (V, T, S, P)$  be the phrase-structure grammar with  $V = \{0, 1, A, S\}$ ,  $T = \{0, 1\}$ , and set of productions  $P$  consisting of  $S \rightarrow 1S$ ,  $S \rightarrow 00A$ ,  $A \rightarrow 0A$ , and  $A \rightarrow 0$ .

- a) Show that 111000 belongs to the language generated by  $G$ .
- b) Show that 11001 does not belong to the language generated by  $G$ .
- c) What is the language generated by  $G$ ?

6. Let  $V = \{S, A, B, a, b\}$  and  $T = \{a, b\}$ . Find the language generated by the grammar ( $V, T, S, P$ ) when the set  $P$  of productions consists of

- a)  $S \rightarrow AB$ ,  $A \rightarrow ab$ ,  $B \rightarrow bb$ .
- b)  $S \rightarrow AB$ ,  $S \rightarrow aA$ ,  $A \rightarrow a$ ,  $B \rightarrow ba$ .
- c)  $S \rightarrow AB$ ,  $S \rightarrow AA$ ,  $A \rightarrow aB$ ,  $A \rightarrow ab$ ,  $B \rightarrow b$ .
- d)  $S \rightarrow AA$ ,  $S \rightarrow B$ ,  $A \rightarrow aaA$ ,  $A \rightarrow aa$ ,  $B \rightarrow bB$ ,  $B \rightarrow b$ .

7. Construct a derivation of  $0^3 1^3$  using the grammar given in Example 5.

24. Let  $G$  be the grammar with  $V = \{a, b, c, S\}$ ;  $T = \{a, b, c\}$ ; starting symbol  $S$ ; and productions  $S \rightarrow abS$ ,  $S \rightarrow bcS$ ,  $S \rightarrow bbS$ ,  $S \rightarrow a$ , and  $S \rightarrow cb$ . Construct derivation trees for

- a) *bcbba*.
- b) *bbbcbba*.
- c) *bcabbbbbcb*.

### 13.2

1. Draw the state diagrams for the finite-state machines with these state tables.

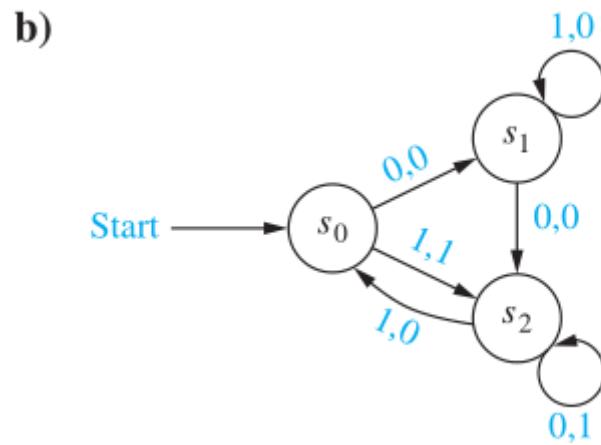
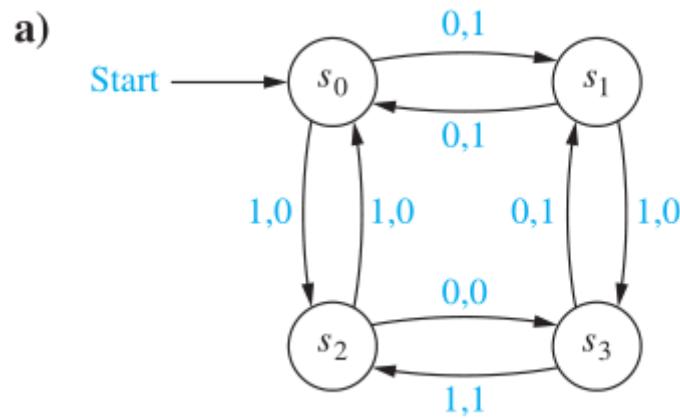
a)

State	<i>f</i>		<i>g</i>	
	<i>Input</i>		<i>Input</i>	
	0	1	0	1
$s_0$	$s_1$	$s_0$	0	1
$s_1$	$s_0$	$s_2$	0	1
$s_2$	$s_1$	$s_1$	0	0

b)

State	<i>f</i>		<i>g</i>	
	<i>Input</i>		<i>Input</i>	
	0	1	0	1
$s_0$	$s_1$	$s_0$	0	0
$s_1$	$s_2$	$s_0$	1	1
$s_2$	$s_0$	$s_3$	0	1
$s_3$	$s_1$	$s_2$	1	0

2. Give the state tables for the finite-state machines with these state diagrams.



3. Find the output generated from the input string 01110 for the finite-state machine with the state table in

- a) Exercise 1(a).
- b) Exercise 1(b).

4. Find the output generated from the input string 10001 for the finite-state machine with the state diagram in

- a) Exercise 2(a).
- b) Exercise 2(b).

5. Find the output for each of these input strings when given as input to the finite-state machine in Example 2.

- a) 0111
- b) 11011011
- c) 01010101010

6. Find the output for each of these input strings when given as input to the finite-state machine in Example 3.

- a) 0000
- b) 101010
- c) 11011100010

### 13.3

1. Let  $A = \{0, 11\}$  and  $B = \{00, 01\}$ . Find each of these sets.

- a)  $AB$
- b)  $BA$
- c)  $A^2$
- d)  $B^3$

5. Describe the elements of the set  $A^*$  for these values of  $A$ .

- a)  $\{10\}$
- b)  $\{111\}$
- c)  $\{0, 01\}$
- d)  $\{1, 101\}$

9. Determine whether the string 11101 is in each of these sets.

- a)  $\{0, 1\}^*$
- b)  $\{1\}^* \{0\}^* \{1\}^*$
- c)  $\{11\} \{0\}^* \{01\}$
- d)  $\{11\}^* \{01\}^*$
- e)  $\{111\}^* \{0\}^* \{1\}$
- f)  $\{11, 0\} \{00, 101\}$

11. Determine whether each of these strings is recognized by the deterministic finite-state automaton in Figure 1.

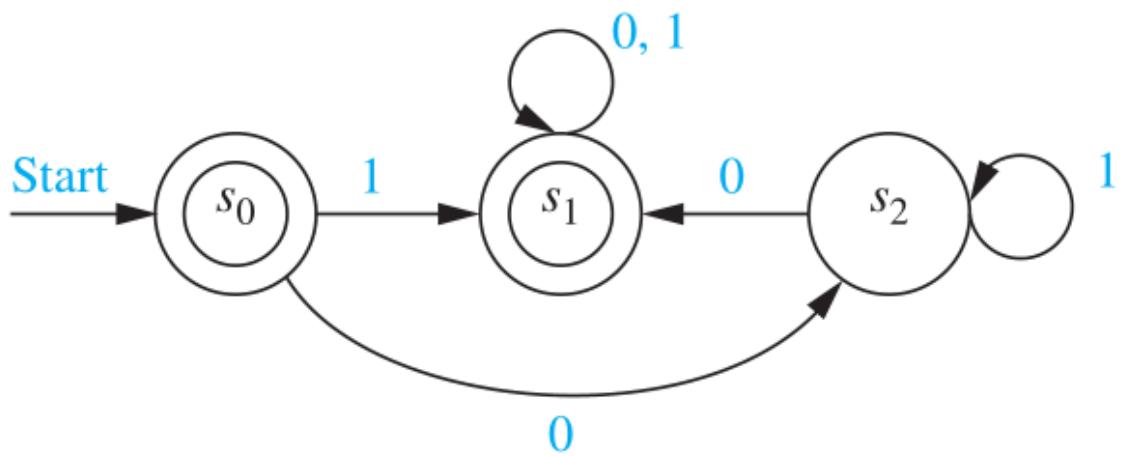
- a) 111
- b) 0011
- c) 1010111
- d) 011011011

12. Determine whether each of these strings is recognized by the deterministic finite-state automaton in Figure 1.

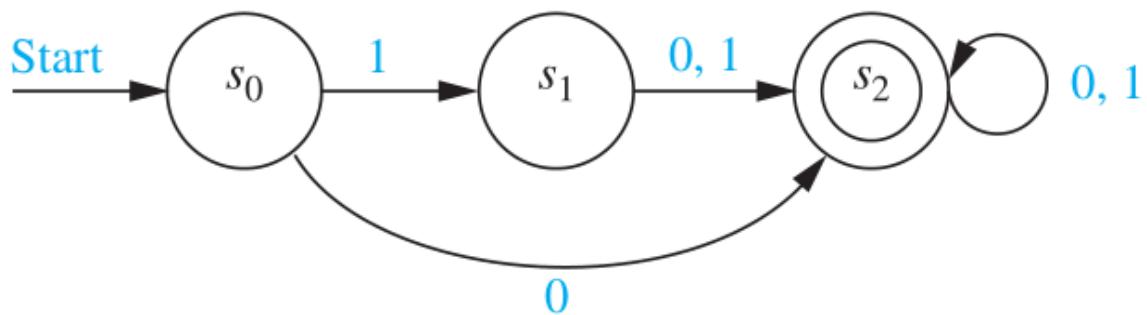
- a) 010
- b) 1101
- c) 1111110
- d) 010101010

In Exercises 16–17 find the language recognized by the given deterministic finite-state automaton.

16.



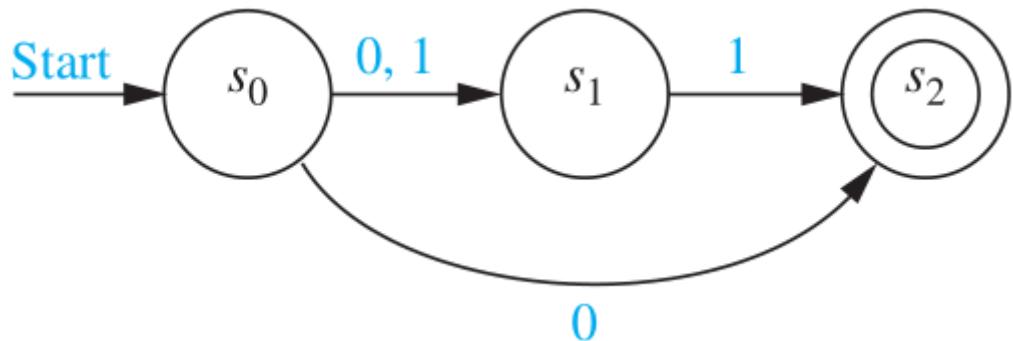
17.



23. Construct a deterministic finite-state automaton that recognizes the set of all bit strings beginning with 01.

24. Construct a deterministic finite-state automaton that recognizes the set of all bit strings that end with 10.

43. find the language recognized by the given nondeterministic finite-state automaton.



50. Find a deterministic finite-state automaton that recognizes the same language as the nondeterministic finite-state automaton in Exercise 43.

#### 13.4

1. Describe in words the strings in each of these regular sets.

- a)  $1 * 0$
- b)  $1 * 00^*$
- c)  $111 \cup 001$

2. Describe in words the strings in each of these regular sets.

- a)  $001^*$
- b)  $(01)^*$
- c)  $01 \cup 001^*$

4. Determine whether 1011 belongs to each of these regular sets.

- a)  $10^*1^*$
- b)  $0^*(10 \cup 11)^*$
- c)  $1(01)^*1^*$

6. Express each of these sets using a regular expression.

- a) the set containing all strings with zero, one, or two bits
- b) the set of strings of two 0s, followed by zero or more 1s, and ending with a 0
- c) the set of strings with every 1 followed by two 0s

#### 13.5

1. Let  $T$  be the Turing machine defined by the five tuples:  $(s0, 0, s1, 1, R)$ ,  $(s0, 1, s1, 0, R)$ ,  $(s0, B, s1, 0, R)$ ,  $(s1, 0, s2, 1, L)$ ,  $(s1, 1, s1, 0, R)$ , and  $(s1, B, s2, 0, L)$ . For each of these initial tapes, determine the final tape when  $T$  halts, assuming that  $T$  begins in initial position.

- a) 

...	$B$	$B$	0	0	1	1	$B$	$B$	...
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- b) 

...	$B$	$B$	1	0	1	$B$	$B$	$B$	...
-----	-----	-----	---	---	---	-----	-----	-----	-----
- c) 

...	$B$	$B$	1	1	$B$	0	1	$B$	...
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3. What does the Turing machine described by the five-tuples  $(s0, 0, s0, 0, R)$ ,  $(s0, 1, s1, 0, R)$ ,  $(s0, B, s2, B, R)$ ,

$(s1, 0, s1, 0, R)$ ,  $(s1, 1, s0, 1, R)$ , and  $(s1, B, s2, B, R)$  do when given

- a) 11 as input?

4.What does the Turing machine described by the five-tuples  $(s0, 0,s0, 1,R)$ ,  $(s0, 1,s0, 1,R)$ ,  $(s0,B, s1,B, L)$ ,  $(s1, 1,s2, 1,R)$ , do when given

a) 101 as input?

5.What does the Turing machine described by the five-tuples  $(s0, 1,s1, 0,R)$ ,  $(s1, 1,s1, 1,R)$ ,  $(s1, 0,s2, 0,R)$ ,  $(s2, 0,s3, 1,L)$ ,  $(s2, 1,s2, 1,R)$ ,  $(s3, 1,s3, 1,L)$ ,  $(s3, 0,s4, 0,L)$ ,  $(s4, 1,s4, 1,L)$ , and  $(s4, 0,s0, 1,R)$  do when given

a) 11 as input?

### Test1

1.The productions of a phrase-structure grammar are  $S \rightarrow S1$ ,  $S \rightarrow 0A$ , and  $A \rightarrow 1$ . Find a derivation of

0111.

2.What language is generated by the phrase-structure grammar if the productions are  $S \rightarrow S11$ ,  $S \rightarrow \lambda$  where  $S$  is the start symbol?

4.Suppose that  $A = 1, 11, 01$  and  $B = 0, 10$ .Find  $AB$  and  $BA$ .

7.Which strings belong to the set represented by the regular expression  $0 * \cup 11$ ?

### Test2

1.What is the language generated by the grammar with productions  $S \rightarrow SA$ ,  $S \rightarrow 0$ ,  $A \rightarrow 1A$ , and  $A \rightarrow 1$ , where  $S$  is the start symbol?

3.Construct a finite-state machine with output that produces a 1 if and only if the last three input bits read are all 0s.

7.Which strings belong to the regular set represented by the regular expression  $(1 * 01 * 0)^*$ ?